RA 25

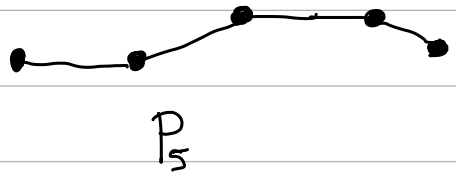
14.3 Properties of Trees

Theorem: There is a unique path between every pair of vertices in a tree.

- A leaf of a free tree is a vertex of degree 1.

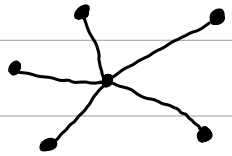
- A vertex is an internal vertex if the vertex has degree at least two.

Theorem: Any free tree with at least 2 vertices has at least two leaves.



P_5

- In general P_n is single path graph that consists of n-vertices.



S_5

- In general S_n is a star graph that consists of a single vertex connected by an edge to k other vertices.

Theorem: Let T be a tree with n vertices and m edges, then $m = n-1$.

- A forest is a graph that has no cycles and is not necessarily connected.

14.4 Tree Traversals

- A tree traversal is a procedure performed on a tree to process the information stored in the vertices by systematically visiting each vertex.

- In a pre-order traversal, a vertex is visited before its descendants.

- Pseudocodes for pre-order and post-order traversals.

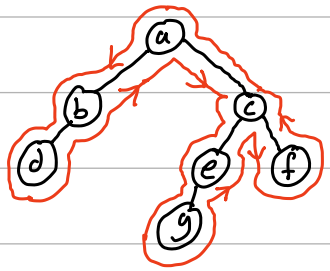
Pre-order

```
Pre-order(v)

process(v)  // generic placeholder for some operation
For every child w of v:
    Pre-order(w)
End-for
```

Post-order

```
Post-order(v)

For every child w of v:
    Post-order(w)
End-for
process(v)
```

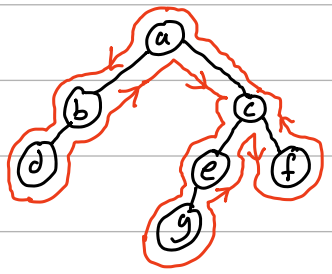
- Useful trick to see order of vertices in pre-order traversal.
  - ↳ Draw an outline all the way around the tree, starting at the root and moving left.
    follow the outline all the way around the tree, when you pass left side of a vertex, visit that vertex.



order-traversed:

a b d c e g f

- Useful trick for post-order traversal:
  - ↳ Same as pre-order but when you pass the right side of a vertex you visit that node.



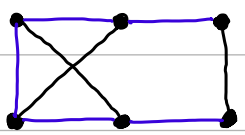
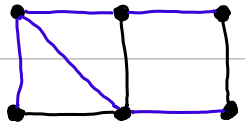
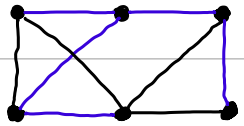
order-traversed:

d b g e f c a

14.5 Spanning trees and graph traversals

- A spanning tree of a connected graph G is a subgraph of G that contains all the vertices in G and is a tree.

- Three different spanning trees of a graph (spanning tree edges in blue)

- Two common methods for finding spanning trees
  - ↳ Breadth-first-search (BFS)
  - ↳ Depth-first-search (DFS)

- Graph traversal is a process that symmetrically explores all the vertices of a graph.